Two-Sided Matching with Indifferences: Using Heuristics to Improve Properties of Stable Matchings

4th Presentation for AS IV of Prof. VESZTEG, Robert Ferenc

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October 27, 2022

Overview

Research Question

Haas (2021)

- GA (Goldberg 1989) - Evolutionary Metaheuristic
- Threshold Accepting algorithm - Local Search Heuristic

1. With respect to what properties, heuristics outperform approximation algorithms of Two-Sided Matching?
2. How is the above statement proved?

Introduction of Two Heuristics

GA (Goldberg 1989) - Evolutionary Metaheuristic

- For complete preferences:
  - Randomly select two genes (matched pairs) of a given chromosome and exchanges either the requester or provider identifiers to create a new chromosome.
  - C1: $\langle$x2,y1$\rangle$,$\langle$x1,y3$\rangle$,$\langle$x3,y2$\rangle$,$\langle$x4,y5$\rangle$
  - $C1^{*}*\langle$x2,y1$\rangle$,$\langle$x1,y3$\rangle$,$\langle$x3,y5$\rangle$,$\langle$x4,y2$\rangle$
- For incomplete preferences:
  - Find Pareto-improvement cycles to increase the number of matched participants.
  - $\langle \varnothing$, y4$\rangle$, $\langle$x1, y3$\rangle$, $\langle$x3, $\varnothing\rangle \rightarrow \langle$x1, y4$\rangle$, $\langle$x3, y3$\rangle$

Introduction of Two Heuristics - Thresholds Accepting algorithm

Threshold Accepting algorithm - Local Search Heuristic

- For complete and incomplete preferences:
  - Given a starting allocation, a set of thresholds;
  - For each of these thresholds, a certain number of adjustments to the allocation are sequentially performed;
  - Compare former properties + threshold to current properties, and take the smaller.
- The combination of the two approaches (GATA): the GA is used to find a good starting allocation for the TA, which then tries to further improve this solution.

Research Results

- Standard set of optimization goals for which approximation algorithms exist (stability and number of matched pairs).
- Matching found by the studied heuristics have improved properties for the respective goal in most of the studied cases.
    1. Average matched rank
       - The overall rank of matched partners averaged over all participants and "satisfaction" with the resulting matching.
    2. Fairness properties
       - The difference of average matched ranks between two sides of the matching market.

How Did the Research Achieve Improved Properties?

Setting of the paper

- Introduction of the setting, brief introduction of past work (most approximation algorithms focus on the mentioned combination of stability and matched pairs).
- Introduction of new measures: egalitarian solutions, including Average Matched Rank and Fairness.
- Empirical simulation of algorithm performance with new measures.
  - Analysis focuses on the AMR and fairness properties of stable matchings found by the different mechanisms.

How Did the Research Achieve Improved Properties?

Setting of the paper

- Individuals of two sides denoted as X and Y, $n_X + n_Y$ in total.
- Individual i of side X has preference profile $P_i = P_{i,j_1}$, ... , P_{i,j_n}, $j \in Y$ over participants of the other side, where $P_{i,j}$ denotes the preference rank that participant i has towards participant j$\in Y \cup \varnothing$.
- Preference profiles are assumed transitive and anti-symmetric.

  Discussions: completeness and incompleteness, existence of indifference
- Goal: to find a matching $\mu = \langle$X, Y$\rangle$ consists of pairs $\langle$x, y$\rangle$ with $x \in X$ and $y \in Y$ that defines which participants are matched together.

Matching Properties

- Blocking pairs
- Number of Matched Pairs
- Average Matched Rank
- Fairness

Matching Property 1. Blocking pairs: Stability

the most fundamental evaluation criterion

- Stability in Two-Sided Matching is defined as the absence of blocking pairs in the allocation. Gale and Shapley (1962)
- DEFINITION. An assignment of applicants to colleges will be called unstable if there are two applicants α and β who are assigned to colleges A and B,respectively, although β prefers A to B, and A prefers β to α.
- DEFINITION. A stable assignment is called optimal if every applicant is at least as well off under it as under any other stable assignment.
- THEOREM. There always exist a stable set of marriages. Irving (1994)
- There is at least one stable matching for a Two-Sided Matching problem, even with incomplete preferences and indifferences.

Matching Property 2. Number of Matched Pairs

Egalitarian Solution

- Complete preferences: always yield the maximum number of matched pairs.
- Incomplete preferences: used as an additional property for the matchings

$$NumPairs = \sum_{<X,Y>} < x, y > |x = \varnothing, y = \varnothing \tag{1}$$

Matching Property 3. Average Matched Rank(AMR)

Egalitarian Solution

- A measure of the average "satisfaction" of participants with the resulting matching.
- Calculated as he overall rank of matched partners averaged over all participants.

$$AMR = \frac{\sum_{i,j \in <X,Y>} P_{i,j} + P_{j,i}}{n_X + n_Y} \tag{2}$$

Matching Property 4. Fairness

Egalitarian Solution

- A measure of difference of average matched rank between the two sides of the matching market.
- Calculated as the difference between two average rank of matched partners.

$$Fairness = \frac{\sum_{i,j \in <X,Y>} P_{i,j}}{n_X} - \frac{\sum_{i,j \in <X,Y>} P_{j,i}}{n_Y}$$

Simulation-Eased Evaluation Approach

Setting

- parameters:
  - n_X and $n_Y = 50$: individuals of side X and Y, population of 50 chromosomes
  - $l = 0.6$: crossover probability, on average, the percentage of participants of the other side are included in a participant's preference list
  - Ψ: the maximum length of the ties in the preference lists
  - ξ: if and to what degree the preferences are correlated
  - a crossover probability of 0.6, and a mutation probability of 0.2 per chromosome were used.

Simulation-Eased Evaluation Approach

Setting

- For complete preferences, the GA uses DA,AMRO, and FE to create initial (stable) solutions (subscript ".DA" in the subsequent evaluations indicates that they are initialized only with DA solutions, whereas ".MIXED" means that a randomly created mixture of DA, AMRO, and FE solutions are used).
- In case of incomplete preferences
  - The baseline version: DA with randomized tie breaking to get the initial population.
  - The mixed version: DA, one matching from each of the approximation algorithms is added to increase the diversity of the initial starting matchings.
  - In total, 1000 evolution rounds are calculated.

Optimizing Stability and Average Matched Rank

Comparison of AMR Performance for Complete Preferences

- Optimizing Stability and Average Matched Rank:
  - Average matched rank becomes worse with an increasing number of participants.
  - GA and GATA are able to improve upon AMRO matchings.

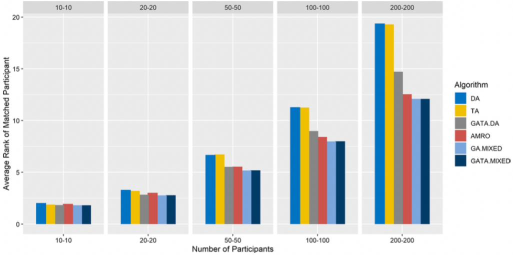
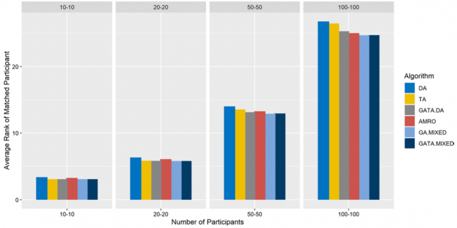


Figure 1: Comparison of AMR properties for complete preferences

Optimizing Stability and Average Matched Rank

Comparison of AMR Performance for Complete and Correlated Preferences

- Optimizing Stability and Average Matched Rank
  - $\xi = 25$: The set of participant is split into two sets of relative size 25-75, highest ranked participants in all the preference profiles are drawn from the same set.
- Average matched rank decreases compared to the uncorrelated case, same relative ranking of the matchings found by different algorithms.



Optimizing Stability and Fairness

Comparison of Fairness Performance for Complete Preferences

- TA finds improvements (in this case: improvements on the fairness performance) only for small problem instances.
- GA.MIXED and GATA.MIXED yield matchings with significantly better fairness properties, and also (slightly) improve upon average FE solutions.

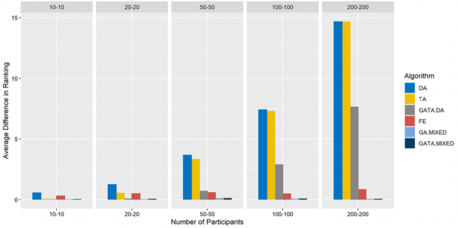


Figure 3: Comparison of AMR performance for complete and correlated preferences

Optimizing Stability and Fairness

Comparison of AMR Performance for Complete and Correlated Preferences

- Optimizing Stability and Average Matched Rank
  - $\xi = 25$: the set of participant is split into two sets of relative size 25-75.
- Matchings found by GA.MIXED and GATA.MIXED seem to have better AMR/fairness properties compared to matchings found by other approaches.

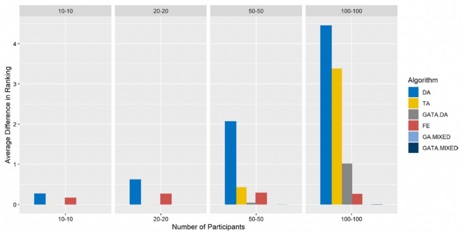


Figure 4: Comparison of AMR performance for complete and correlated preferences

Comparing Matching Properties

Incomplete Preferences With Indifferences

- For all considered scenarios the algorithms are within 95% of the optimal solution.

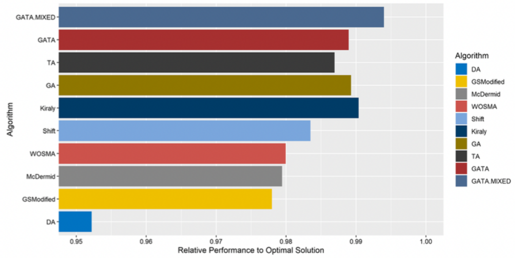


Figure 5: Algorithm comparison relative to optimum, uncorrelated preferences, 10 $\times$ 10 to 100 $\times$ 100 participants

Comparing Matching Properties

Incomplete Preferences With Indifferences

- The heuristics not only perform well on average, they also find matchings with the optimal percentage of matched participants in more cases than the approximation algorithms.

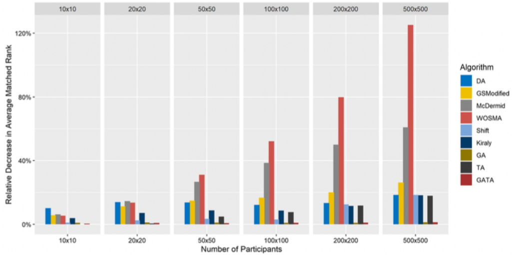


Figure 6: Relative decrease in average matched rank (AMR) compared to GATA.MIXED

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